Nominal Rigidities *à* la Rotemberg

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1 New Keynesian Phillips Curve (NKPC)

The problem of each intermediate producer j is to maximize real profits subject to quadratic adjustment costs in units of the final good

$$\max_{P_t(j)} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j) \right) - \frac{\zeta_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \right],$$

subject to

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t \quad \text{and} \quad Y_t(j) = A_t N_t(j)^{1-\alpha}.$$

The first order condition is

$$\frac{1}{P_t} \left(Y_t(j) + P_t(j) \frac{\partial Y_t(j)}{\partial P_t(j)} \right) - \frac{W_t}{P_t} \frac{\partial N_t(j)}{\partial Y_t(j)} \frac{\partial Y_t(j)}{\partial P_t(j)} - \zeta_p \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} \\ + \beta \zeta_p \mathbb{E}_t \left[\left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{P_t(j)^2} Y_{t+1} \right] = 0.$$

Noting that

$$\frac{\partial Y_t(j)}{\partial P_t(j)} = -\varepsilon_p \frac{Y_t(j)}{P_t(j)} \quad \text{and} \quad \frac{\partial N_t(j)}{\partial Y_t(j)} = \frac{N_t(j)}{(1-\alpha) Y_t(j)},$$

the optimality condition under a symmetric equilibrium ($P_t(j) = P_t$, which implies $Y_t(j) = Y_t$ and $N_t(j) = N_t$) reads as

$$(\Pi_{p,t} - 1)\Pi_{p,t} = \frac{\varepsilon_p}{\zeta_p} \left(RMC_t - \frac{\varepsilon_p - 1}{\varepsilon_p} \right) + \beta \mathbb{E}_t \left((\Pi_{p,t+1} - 1)\Pi_{p,t+1} \frac{Y_{t+1}}{Y_t} \right),$$

where $\Pi_{p,t} = P_t / P_{t-1}$.

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2 New Keynesian Wage Phillips Curve (NKWPC)

We consider the problem of a union determining the optimal wage for task i. We consider a version in which we have two agents and there is a quadratic utility cost of change wages across periods. Formally, the union sets the optimal wage that maximizes the (average) lifetime utility of household(s), subject to the adjustment cost, the demand for labor and the budget constraint

$$\max_{W_t(i)} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\lambda U(C_t^c, N_t^c) + (1-\lambda) U(C_t^u, N_t^u) - \frac{\zeta_w}{2} \left(\frac{W_t(i)}{W_{t-1}(i)} - 1 \right)^2 \right]$$

subject to

$$N_t(i) = \left(\frac{W_t(i)}{W_t} \right) N_t \quad \text{and} \quad C_t^h = \int_0^1 \frac{W_t(i) N_t(i)}{P_t} di, \;\; \text{for } h = c, u,$$

and where the utility function is given by

$$U(C_t^h, N_t^h) = \frac{(C_t^h)^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di.$$

Assuming the union sets the same wage/hours for each household ($N_t^h(i) = N_t(i)$, for every h), the first order condition is

$$\lambda (C_t^c)^{-\sigma} \frac{\partial C_t^c}{\partial W_t(i)} + (1-\lambda)(C_t^u)^{-\sigma} \frac{\partial C_t^u}{\partial W_t(i)} - \chi N_t(i)^{\varphi} \frac{\partial N_t(i)}{\partial W_t(i)} - \zeta_w \left(\frac{W_t(i)}{W_{t-1}(i)} - 1 \right) \frac{1}{W_{t-1}(i)} + \beta \zeta_w \mathbb{E}_t \left[\left(\frac{W_{t+1}(i)}{W_t(i)} - 1 \right) \frac{W_{t+1}(i)}{W_t(i)} \right] = 0.$$

Noting that the relevant partial derivatives read as

$$\frac{\partial C_t^h}{\partial W_t(i)} = \frac{(1-\varepsilon_w) N_t(i)}{P_t} \quad \text{and} \quad \frac{\partial N_t(i)}{\partial W_t(i)} = -\varepsilon_w \frac{N_t(i)}{W_t},$$

the first order condition under a symmetric equilibrium reads as

$$(\Pi_{w,t} - 1)\Pi_{w,t} = \frac{\varepsilon_w}{\zeta_w} N_t \left[\chi N_t^{\varphi} - \left(\frac{\varepsilon_w - 1}{\varepsilon_w} \right) \overline{U}_{c,t} w_t \right] + \beta \mathbb{E}_t \Big((\Pi_{w,t+1} - 1)\Pi_{w,t+1} \Big),$$

with $\overline{U}_{c,t} = \lambda (C_t^c)^{-\sigma} + (1-\lambda)(C_t^u)^{-\sigma}$ and $w_t = W_t / P_t$.